ON THE FORM OF OHM'S LAW IN MAGNETOHYDRODYNAMICS

(O FORME ZAKONA OMA V MAGNITNOI GIDRODINAMIKE)

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To describe the motion of an electrically conducting fluid or gas in an electromagnetic field one uses the system of mechanical equations and the system of Maxwell's equations, which in the case of interaction of the field with the medium must be considered simultaneously. This combined system of equations is complete (that is, it suffices to determine all quantities characterizing the motion of the medium and the variation of the electromagnetic field) if expressions are given for the internal energy of the medium, the stress tensor, and the heat-flux vector (the expressions for these quantities are usually taken from ordinary hydrodynamics; refinements for an ionized gas moving in a magnetic field can be found in [1]), and if the connection is also given between the current density $\mathbf{j}$ and the other quantities characterizing the problem. As a simple relation for the current density in electrodynamics and magnetohydrodynamics one uses Ohm's law (see, for example, [2] and [3])

$$\mathbf{j}' = \sigma \mathbf{E}' \tag{0.1}$$

Here $\mathbf{j}$ is the current density, $\mathbf{E}$ the intensity of the electric field, and σ is a coefficient called the conductivity of the medium; primes mean that the corresponding quantities are taken in a system of coordinates in which the medium is at rest. If one uses the formulas for transforming the field and current from one system of coordinates to another, the relation (0.1) can be written in a system of coordinates in which the motion of the medium is considered as

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{H}\right) + \rho_e \mathbf{v} \tag{0.2}$$

Here ρ_e is the electric charge density, $\mathbf{v}$ the velocity of the medium, and c a constant equal to the speed of light. Here, and throughout what follows, we use an absolute Gaussian system of units. Relations (0.1) and (0.2) describe well the flow of current in solid conductors and also in liquid and gaseous conductors for sufficient density of the media and moderate magnetic fields, but describe poorly the process of current flow in a number of other cases that are of considerable present interest. In this connection a number of authors have considered the question of the form of a so-called generalized Ohm's law suitable for a description of the phenomena of current flow in a fully ionized gas [4,5] or a partially ionized gas [6] under conditions in which the relations (0.1) and (0.2) are unsuitable.

To obtain a generalized Ohm's law it is convenient to use a model multi-component quasi-neutral medium consisting of electrons, ions, and neutral atoms [4,6]. In the present paper there is obtained on the basis of this model a relation connecting the current density with the other parameters (generalized Ohm's law) in the presence of a certain space charge ρ_e. Furthermore, hypotheses and conjectures are formulated and discussed that specify the form of the generalized Ohm's law, and consequently also the limits of applicability of the relations obtained. (In particular, the range of the hypotheses is indicated in which relations (0.1) and (0.2) are satisfied.) Dimensionless parameters are given, related to the mechanical and physical characteristics of the problem, that determine the form of Ohm's law. Various forms of Ohm's law are discussed from the point of view of their application to one or another concrete problem.

1. General equations describing the motion of a three-component medium consisting of electrons, ions and neutral atoms. In order to obtain a relation connecting the current density with the quantities characterizing the motion of the medium and the electromagnetic field intensity, and which is independent of Maxwell's equations and the equations of mechanics, we consider a simplified kinetic molecular model of the medium.

Let a unit volume of the medium contain n_a neutral atoms, n ions, and $n + n'$ electrons. For simplicity we assume that the electrons and ions carry equal but opposite charge e; then the space charge density is equal to

$$\rho_e = -n'e$$

Furthermore, we assume that the mass m_i of an ion is much larger than the mass m_e of an electron, and equal to the mass m_a of a neutral atom.

The degree of ionization is the quantity

$$\alpha = n / (n + n_a) \tag{1.1}$$

As a simplified model we suppose that each component – electrons, ions, and neutral atoms – represents a gas moving independently of the other components, in the sense that the hydrodynamic equations of motion may be written separately for each component. Interaction between the components arises as a result of collisions of particles, and leads to a certain mean force equal to the average change of momentum due to collisions of particles belonging to the different components.

The electron, ion and neutral gases are regarded as ideal, so that the stress within the components amounts to appropriate pressures. (See [7] for the effect upon Ohm's law of terms related to the viscosity of the components.)

If the velocities of relative motion of the components are small compared with the random speed of the particles, then the mixture as a whole may also be considered an ideal fluid, and the total pressure is equal to the sum of the partial pressures of each component (see [1], for example)

$$p = p_e + p_i + p_a \tag{1.2}$$

Since under equilibrium conditions the pressure is proportional to the number of particles, the following formulas are valid:

$$p_i = \frac{n}{2n + n_a + n'}\,p, \qquad p_e = \frac{n + n'}{2n + n_a + n'}\,p, \qquad p_a = \frac{n_a}{2n + n_a + n'}\,p \tag{1.3}$$

In the space occupied by the moving medium the electric field $\mathbf{E}$ and magnetic field $\mathbf{H}$ are given. Here it is assumed that $\mathbf{E}$ and $\mathbf{H}$ are determined as external fields, as are also the charges and currents in the medium itself.

We determine the force acting upon each component that arises from collision of particles of the given component with particles of another component. The force acting on the νth component due to the kth component can be represented in the following form:

$$\mathbf{f}_{\nu k} = \Delta \mathbf{J}_{\nu k} n_\nu \tau_{\nu k}^{-1}, \qquad \nu, k = e, i, a \tag{1.4}$$

Here n_ν is the number of particles of the νth kind per unit volume, $\tau_{\nu k}$ the average time between collisions of particles of the νth kind with particles of the kth kind, where the average time between collisions is taken as the interval of time until a particle of the νth kind loses on the average an impulse $\Delta \mathbf{J}_{\nu k}$ by interaction with a particle of the kth kind.

Usually one takes as $\Delta \mathbf{J}_{\nu k}$ the quantity

$$\Delta \mathbf{J}_{\nu k} = -\frac{m_k m_\nu}{m_k + m_\nu} \mathbf{v}_{\nu k}$$

Here $\mathbf{v}_{\nu k}$ is the average velocity of particles of the νth component relative to particles of the kth component, which corresponds to the average loss of momentum for elastic collision of two particles of masses m_ν and m_k moving with relative velocity $\mathbf{v}_{\nu k}$, under the assumption that all angles of deflection of the particle as a result of the collision are equally probable.

Since $m_i \gg m_e$, for the electron gas

$$\Delta \mathbf{J}_{ek} = -\mathbf{J}_{ek} \qquad (k = i,\ a)$$

and for the ion and neutral gases

$$\Delta \mathbf{J}_{\nu k} = -\frac{1}{2} \mathbf{J}_{\nu k} \qquad (k,\ \nu = i,\ a;\ k \neq \nu)$$

Here $\mathbf{J}_{\nu k}$ is the impulse of particles of the νth gas relative to particles of the kth.

It is now easy to write the equations of motion for each component of the medium. Let the velocity of motion of the entire medium be $\mathbf{v}$, the velocity of motion of the ion gas relative to the medium be $\mathbf{v}_i$, and the velocity of motion of the electron gas relative to the ion gas be $\mathbf{v}_e$. The velocity of motion of the neutral gas is determined from the velocity of the medium and the velocity of the ion gas by the formula

$$\mathbf{v}_a = \mathbf{v} - \frac{n}{n_a}\left(\mathbf{v}_i + \frac{m_e}{m_i}\mathbf{v}_e\right) \tag{1.5}$$

In the derivation of this relation it is assumed that the velocity of an element of the medium coincides with the velocity of its mass center, and the inequality $m_e \ll m_i$ is also used. The last term is retained in spite of the small mass of the electron because of the fact that the relation between $\mathbf{v}_e$ and $\mathbf{v}_i$ is unknown, and it may happen that $|m_e\mathbf{v}_e| \approx |m_i\mathbf{v}_i|$.

Furthermore, it is assumed here and henceforth that $n' \ll n$, because significant concentrations of space charge cannot arise in the absence of special outer conditions providing for the retention of this charge. In this connection terms of order n'/n are neglected everywhere in comparison with terms of order unity.

We note that the condition $n' \ll n$ is not equivalent to the assumption of the absence of space charge, because a small excess in the number of electrons over the number of ions can give a marked increase in the force due to the electric field ($-n'eE = \rho_e E$), and in the current density due to the transfer of charges together with the motion of the medium ($-n'ev = \rho_e v$). If the gas is fully ionized ($n_a = v_a = 0$), relation (1.5) simplifies and gives a connection between v_i and v_e. Calculations analogous to those carried out here under the assumption that $\alpha \neq 1$ ($n_a \neq 0$) can be carried out also for the case of a fully ionized (two-component) medium [7], where, as is easily verified, the generalized Ohm's law can be obtained from the relation (2.12) of the present paper in the limit $\alpha \to 1$ (relation (2.13)).

In addition to the forces due to collisions of particles of the different components, a force due to the electromagnetic field will act on the electron and ion gases.

The average momentum of electrons relative to ions is

$$\mathbf{J}_{ei} = m_e \mathbf{v}_e$$

The average momentum of electrons relative to the neutral molecules is

$$\mathbf{J}_{ea} = m_e (\mathbf{v} + \mathbf{v}_i + \mathbf{v}_e - \mathbf{v}_a)$$
$$= m_e\Big[\mathbf{v}_e + \Big(1 + \frac{n}{n_a}\Big)\mathbf{v}_i + \frac{n}{n_a}\frac{m_e}{m_i}\mathbf{v}_e\Big] = m_e\Big(\mathbf{v}_e + \frac{\mathbf{v}_i}{1-\alpha}\Big)$$

Consequently the equation of motion for the electron gas may be written in the form

$$m_e n \frac{d_e(\mathbf{v} + \mathbf{v}_i + \mathbf{v}_e)}{dt} = -\operatorname{grad} p_e - ne\Big[\mathbf{E} +$$
$$+ \frac{1}{c}(\mathbf{v} + \mathbf{v}_i + \mathbf{v}_e) \times \mathbf{H}\Big] - m_e n \mathbf{v}_e \tau^{-1} - m_e n\Big(\mathbf{v}_e + \frac{\mathbf{v}_i}{1-\alpha}\Big)\tau_e^{-1} \qquad (1.6)$$
$$\frac{d_e}{dt} = \frac{\partial}{\partial t} + [(\mathbf{v} + \mathbf{v}_i + \mathbf{v}_e)\nabla] = \frac{d}{dt} + (\mathbf{v}_i + \mathbf{v}_e)\nabla$$

The momentum of the ions relative to the neutral atoms is equal to

$$\mathbf{J}_{ia} = m_i(\mathbf{v} + \mathbf{v}_i - \mathbf{v}_a) = m_i\frac{\mathbf{v}_i}{1-\alpha} + m_e\frac{\alpha}{1-\alpha}\mathbf{v}_e$$

Using this expression, we write the equation of motion of the ion gas in the form

$$m_i n \frac{d_i(\mathbf{v} + \mathbf{v}_i)}{dt} = -\operatorname{grad} p_i + ne\Big[\mathbf{E} + \frac{1}{c}(\mathbf{v} + \mathbf{v}_i) \times \mathbf{H}\Big] + nm_e\mathbf{v}_e\tau^{-1} -$$
$$-\frac{1}{2}nm_i\frac{\alpha}{1-\alpha}\mathbf{v}_e\tau_i^{-1}\frac{m_e}{m_i} - \frac{1}{2}nm_i\frac{\mathbf{v}_i}{1-\alpha}\tau_i^{-1} \qquad \Big(\frac{d_i}{dt} = \frac{\partial}{\partial t} + (\mathbf{v} + \mathbf{v}_i)\nabla\Big) \quad (1.7)$$

In Equations (1.6) and (1.7) τ, τ_i, and τ_e are respectively the times between collisions of electrons and ions, ions and neutral atoms, and electrons and neutral atoms.

In place of the equation of motion of the neutral gas we will use the equation of motion for the medium as a whole, which is, of course, a consequence of the equation of motion of the neutral gas and of Equations (1.6) and (1.7):

$$m_i (n + n_a) \frac{d\mathbf{v}}{dt} = - \operatorname{grad} p - n'e\mathbf{E} - \frac{e}{c} [n\mathbf{v}_e + n' (\mathbf{v} + \mathbf{v}_i)] \times \mathbf{H} \quad (1.8)$$

Here were used for the determination of the current density

$$\mathbf{j} = \sum_k n_k e_k \mathbf{v}_k = - (n + n') e (\mathbf{v} + \mathbf{v}_i + \mathbf{v}_e) + $$
$$+ ne (\mathbf{v} + \mathbf{v}_i) = - ne\mathbf{v}_e - n'e (\mathbf{v} + \mathbf{v}_i)$$

and the inequality $n' \ll n$.

2. Resulting generalized Ohm's law. In Equations (1.6), (1.7) and (1.8) we estimate the terms containing derivatives.

We will assume that the characteristic time of the problem is much larger than the time between collisions, and that the velocity of the components relative to the center of mass is small compared with the random speed of the particles belonging to a given component. (This condition was used in obtaining relations (1.2) and (1.3).) If T is a characteristic time of the problem, L a characteristic dimension, and U a characteristic speed ($U = L/T$), then these assumptions are equivalent to the following:

$$T \gg \max\{\tau, \tau_e, \tau_i\}$$
$$v_i \ll v_{ix}, \qquad |\mathbf{v}_i + \mathbf{v}_e| \ll v_{ex}, \qquad |\mathbf{v} - \mathbf{v}_a| < v_{ax} \quad (2.1)$$

Here v_{ix}, v_{ex}, and v_{ax} are the chaotic speeds of the ions, electrons and neutral particles; hence, if there is a state of equilibrium, the temperature of the electrons, ions and neutral particles is equal, and

$$m_e v_{ex}^2 = m_i v_{ix}^2 = m_a v_{ax}^2$$

If the conditions (2.1) do not hold, it is possible to obtain from Equations (1.6) to (1.8) a single relation connecting current density with the other parameters in the problem only under sufficiently special assumptions, for example if the condition $v_i \ll v$ holds.

Because the pressure is proportional to the product of the mass of the particles and the mean square random speed, under conditions (2.1) the terms

$$\frac{d}{dt}(\mathbf{v}_i + \mathbf{v}_e), \qquad (\mathbf{v}_i + \mathbf{v}_e)\nabla(\mathbf{v} + \mathbf{v}_i + \mathbf{v}_e), \qquad \frac{d\mathbf{v}_i}{dt}, \ \mathbf{v}_i\nabla(\mathbf{v} + \mathbf{v}_i)$$

on the left sides of Equations (1.6) and (1.7) can be neglected compared with the last terms on the right sides of the equations and with the gradients of the corresponding pressures.

Furthermore, in view of Equation (1.8) and Formula (1.3) the following relation holds under the conditions $m_i \gg m_e$, $m_i = m_a$, $n \gg n'$:

$$m_e n\left|\frac{d\mathbf{v}}{dt}\right| = \frac{m_e}{m_i}\frac{n}{n+n_a}\left|\left[-n'e\mathbf{E} + \operatorname{grad} p - \frac{ne}{c}(\mathbf{v}_e \times \mathbf{H}) - \right.\right.$$
$$\left.\left. - n'e(\mathbf{v} + \mathbf{v}_i) \times \mathbf{H}\right]\right| \ll \left|-\operatorname{grad} p_e - ne\mathbf{E} - \frac{ne}{c}(\mathbf{v} + \mathbf{v}_i + \mathbf{v}_e) \times \mathbf{H}\right|$$

That is, the term $m_e n\, d\mathbf{v}/dt$ in the left side of Equation (1.6) can also be neglected. Thus, under conditions (2.1), Equations (1.6) to (1.8) take the form

$$-\operatorname{grad} p_e - ne\left[\mathbf{E} + \frac{1}{c}(\mathbf{v} + \mathbf{v}_i + \mathbf{v}_e) \times \mathbf{H}\right] - m_e n \mathbf{v}_e \tau^{-1} -$$
$$- m_e n\left(\mathbf{v}_e + \frac{\mathbf{v}_i}{1-\alpha}\right)\tau_e^{-1} = 0$$
$$m_i n\frac{d\mathbf{v}}{dt} = -\operatorname{grad} p_i + ne\left[\mathbf{E} + \frac{1}{c}(\mathbf{v} + \mathbf{v}_i) \times \mathbf{H}\right] + nw_e\mathbf{v}_e\tau^{-1} - \qquad (2.2)$$
$$-\frac{1}{2}nm_i\frac{\alpha}{1-\alpha}\mathbf{v}_e\tau_i^{-1}\frac{m_e}{m_i} - \frac{1}{2}nm_i\frac{\mathbf{v}_i}{1-\alpha}\tau_i^{-1}$$
$$m_i(n + n_a)\frac{d\mathbf{v}}{dt} = -\operatorname{grad} p - n'e\mathbf{E} - \frac{e}{c}[n\mathbf{v}_e + n'(\mathbf{v} + \mathbf{v}_i)] \times \mathbf{H}$$

We introduce the terminology

$$\mathbf{j} = -ne\mathbf{v}_e - n'e(\mathbf{v} + \mathbf{v}_i), \ \mathbf{j}_i = ne\mathbf{v}_i$$

$$\varkappa \equiv \frac{1}{\omega_e\tau} = \frac{cm_e}{eH}\tau^{-1}, \qquad \varkappa_e \equiv \frac{1}{\omega_e\tau_e} = \frac{cm_e}{eH}\tau_e^{-1}, \ \varkappa_i \equiv \frac{1}{\omega_i\tau_i} = \frac{1}{2}\frac{cm_i}{eH}\tau_i^{-1} \quad (2.3)$$

Here ω_e and ω_i are the Larmor frequencies of the electron and ion. Thus the quantities $\omega_e\tau$, $\omega_e\tau_e$, and $\omega_i\tau_i$ denote the number of turns of the spiral trajectory that the corresponding particle executes in the time between two collisions with particles of the other components.

With the notation (2.3) and under the conditions $m_i \gg m_e$ and $n \gg n'$, Equations (2.2) take the form

$$
\begin{gathered}
-\operatorname{grad} p_e - en\Big(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\Big) - \frac{1}{c}\,\mathbf{j}_i\times\mathbf{H} + \frac{1}{c}\,\mathbf{j}\times\mathbf{H} + \\
+ \frac{1}{c}(\varkappa + \varkappa_e)\,H\,(\mathbf{j} + n'e\mathbf{v}) - \Big(\frac{H}{1-\alpha}\frac{\varkappa_e}{c} - \frac{\varkappa}{c}H\frac{n'}{n}\Big)\mathbf{j}_i = 0
\end{gathered} \tag{2.4}
$$

$$
\begin{gathered}
-\operatorname{grad} p_i + ne\Big(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\Big) + \frac{1}{c}\,\mathbf{j}_i\times\mathbf{H} - \Big(\frac{\varkappa}{c}H - \\
- \frac{\alpha}{1-\alpha}\frac{m_e}{m_i}\frac{\varkappa_i}{c}H\Big)(n'e\mathbf{v} + \mathbf{j}) - \Big(\frac{\varkappa}{c}H\frac{n'}{n} + \frac{H}{1-\alpha}\frac{\varkappa_i}{\alpha}\Big)\mathbf{j}_i = nm_i\frac{d\mathbf{v}}{dt}
\end{gathered} \tag{2.5}
$$

$$
-\operatorname{grad} p + \frac{1}{c}\,\mathbf{j}\times\mathbf{H} - n'e\mathbf{E} = \frac{m_i n}{\alpha}\frac{d\mathbf{v}}{dt} \tag{2.6}
$$

Combining (2.4) and (2.5) and eliminating $d\mathbf{v}/dt$ with the aid of (2.6), we obtain the expression

$$
\begin{gathered}
\mathbf{j}_i = \frac{(1-\alpha)\,c}{H(\varkappa_e + \varkappa_i)}\Big[\alpha \operatorname{grad} p - \operatorname{grad}(p_e + p_i) - (1-\alpha)\,n'e\,\mathbf{E} + \\
+ n'e\mathbf{v}\frac{H}{c}\Big(\varkappa_e + \frac{\alpha}{1-\alpha}\frac{m_e}{m_i}\varkappa_i\Big) + \frac{(1-\alpha)}{c}\,\mathbf{j}\times\mathbf{H} + \frac{H}{c}\Big(\frac{\alpha}{1-\alpha}\frac{m_e}{m_i}\varkappa_i + \varkappa_e\Big)\mathbf{j}\Big]
\end{gathered} \tag{2.7}
$$

(In the sum of Equations (2.4) and (2.5) it is necessary to take into account the term $n'eE$, because the terms $\pm ne\mathbf{E}$ appearing in these equations cancel in the combination.)

Now eliminating $\mathbf{j}_i$ from Equation (2.4) with the aid of (2.7), we obtain a relation connecting the current density with the electromagnetic-field intensity and the parameters characterizing the medium and its motion

$$
\begin{gathered}
-\left[\operatorname{grad} p_e + \beta\,(\alpha \operatorname{grad} p - \operatorname{grad}(p_i + p_e))\right] - ne\Big(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\Big) + \\
+ \Big[1 - 2(1-\alpha)\,\beta - \alpha\,\frac{\varkappa_i}{\varkappa_e + \varkappa_i}\frac{m_e}{m_i}\Big]\frac{1}{c}\,\mathbf{j}\times\mathbf{H} + \beta\alpha n'e\mathbf{E} - \\
- \frac{1-\alpha}{c}\Big(\beta + \frac{\alpha}{1-\alpha}\frac{m_e}{m_i}\frac{\varkappa_i}{\varkappa_e + \varkappa_i}\Big)en'\mathbf{v}\times\mathbf{H} + \Big[\varkappa + (1-\beta)\,\varkappa_e - \\
- \beta\frac{\alpha}{1-\alpha}\frac{m_e}{m_i}\varkappa_i\Big]\frac{H}{c}(\mathbf{j} + n'e\mathbf{v}) - \frac{1-\alpha}{H(\varkappa_e + \varkappa_i)}\Big\{[\alpha \operatorname{grad} p - \\
- \operatorname{grad}(p_e + p_i)]\times\mathbf{H} + \frac{1-\alpha}{c}\,\mathbf{j}\times\mathbf{H}\times\mathbf{H} - (1-\alpha)\,n'e\mathbf{E}\times\mathbf{H}\Big\} = \\
= -\varkappa\,\frac{n'}{n}\frac{1-\alpha}{\varkappa_e + \varkappa_i}\Big[\alpha \operatorname{grad} p - \operatorname{grad}(p_e + p_i) - (1-\alpha)\,n'e\mathbf{E} + \\
+ (\mathbf{j} + n'e\mathbf{v})\,\frac{H}{c}\Big(\varkappa_e + \frac{\alpha}{1-\alpha}\frac{m_e}{m_i}\varkappa_i\Big) + \frac{1-\alpha}{c}\,\mathbf{j}\times\mathbf{H}\Big]
\end{gathered} \tag{2.8}
$$

$$
\Big(\beta = \frac{\varkappa_e}{\varkappa_i + \varkappa_e}\Big)
$$

Relation (2.8) may be regarded as the generalized form of Ohm's law for a partially ionized gas.

If the speed of random motion is large compared with the relative speeds of motion of the components, the time between collisions is determined by the random speeds (v_{ex}, v_{ix}). In equilibrium the electrons and ions possess equal kinetic energies of random motion, but the mean free path of the ion between its collisions with neutral atoms (l_{ia}) is less than the mean free path of the electron between its collisions with neutral atoms (l_{ea}), so that

$$\frac{\tau_i}{\tau_e}=\frac{v_{ex}}{l_{ea}}\frac{l_{ia}}{v_{ix}}\approx\sqrt{\frac{m_i}{m_e}}\frac{l_{ia}}{l_{ea}}<\sqrt{\frac{m_i}{m_e}}$$

In view of the condition $m_i \gg m_e$ it follows that

$$\frac{\varkappa_e}{\varkappa_i}=\frac{\omega_i\tau_i}{\omega_e\tau_e}\approx\frac{m_e}{m_i}\frac{\tau_i}{\tau_e}<\frac{m_e}{m_i}\sqrt{\frac{m_i}{m_e}}\ll 1 \tag{2.9}$$

That is, between two collisions with neutral atoms the electrons execute significantly more turns of the spiral trajectory than do the ions. Here $\beta \ll 1$ and relation (2.8) simplifies to

$$\begin{aligned}
&-\operatorname{grad} p_e - ne\Big(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H}\Big)+\frac{1}{c}\,\mathbf{j}\times\mathbf{H}+\frac{\varkappa+\varkappa_e}{c}H(\mathbf{j}-\mathbf{v}\rho_e)-\\
&-\frac{1-\alpha}{\varkappa_i H}\Big\{[\alpha\operatorname{grad} p-\operatorname{grad}(p_e+p_i)]\times\mathbf{H}+\frac{(1-\alpha)}{c}\,\mathbf{j}\times\mathbf{H}\times\mathbf{H}+\\
&+(1-\alpha)\,\rho_e\mathbf{E}\times\mathbf{H}\Big\}=-\frac{\varkappa}{\varkappa_i}\frac{n'}{n}(1-\alpha)\Big[\alpha\operatorname{grad} p-\operatorname{grad}(p_e+p_i)+\\
&\qquad+(1-\alpha)\,\rho_e\mathbf{E}+\frac{1-\alpha}{c}\,\mathbf{j}\times\mathbf{H}\Big]
\end{aligned} \tag{2.10}$$

On the right side of Equation (2.10) some small terms have been deleted by virtue of $m_i \gg m_e$, $n \gg n'$ and (2.9).

Terms appearing in the right side of Equation (2.10) can, by virtue of $n \gg n'$, be neglected compared with corresponding terms on the left side of this equation if the condition is satisfied

$$\frac{\varkappa}{\varkappa_i}=\frac{\omega_i\tau_i}{\omega_e\tau}\approx\frac{m_e}{m_i}\frac{\tau_i}{\tau}<\sqrt{\frac{m_e}{m_i}}\frac{\tau_e}{\tau}\lesssim 1 \tag{2.11}$$

The time τ_e between collisions of an electron with neutral atoms is larger than the time τ between collisions of the electron with ions, thanks to the remote collisions between charged particles (see [4], for example). For gases that are not highly rarefied at moderate temperature, the relation (2.11) holds, and the generalized Ohm's law has the form

$$-\operatorname{grad} p_e - ne\Big(\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H}\Big)+\frac{1}{c}\,\mathbf{j}\times\mathbf{H}+\frac{\varkappa+\varkappa_e}{c}H(\mathbf{j}-\rho_e\mathbf{v})-$$

$$-\frac{(1-\alpha)^2}{H\varkappa_i}\left\{-\frac{\alpha}{\alpha+1}\operatorname{grad} p \times \mathbf{H} + \frac{1}{c}\,\mathbf{j}\times\mathbf{H}\times\mathbf{H} + \rho_e\mathbf{E}\times\mathbf{H}\right\} = 0 \qquad (2.12)$$

In order to simplify the subsequent equations, the first term of the last item is transformed for the cases n = const, n_a = const.

If the gas is completely ionized ($a = 1$), $\kappa_e = \kappa_i = 0$. Furthermore, it follows from (1.5) that $\mathbf{j}_i = (m_e/m_i)\mathbf{j}$, that is, as $a \to 1$, the quantities κ_i, κ_e, and $(1-a)^2/\kappa_i$ tend to zero, and Ohm's law takes the form

$$-\operatorname{grad} p_e - ne\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right) + \frac{1}{c}\,\mathbf{j}\times\mathbf{H} + \frac{\varkappa}{c}H(\mathbf{j}-\rho_e\mathbf{v}) = 0 \quad (2.13)$$

Henceforth the inequality (2.11) is always assumed to be satisfied. If this inequality is violated in a concrete case, it is necessary to take into account in Ohm's law terms in the right side of (2.10).

3. Different forms of the generalized Ohm's law. The magnitudes of the coefficients in Equation (2.12) depend on the physical properties of the medium under consideration. Furthermore, the magnitudes of the individual terms in this equation depend upon the mechanical characteristics (velocity, pressure, etc.) of the problem under consideration and the magnitude of the electromagnetic-field intensity. In this connection, it can be shown in one or another specific problem that certain terms in Equation (2.12) are negligibly small. Thus it is possible to use a simpler form of the generalized Ohm's law. In order to ascertain which parameters determine the form of the generalized Ohm's law, we estimate the relative magnitudes of the terms appearing in Equation (2.12).

We observe first of all that if the gas is partially ionized, and

$$\frac{(1-\alpha)^2}{\varkappa_i} \ll 1 \qquad (3.1)$$

then

$$\frac{(1-\alpha)^2}{\varkappa_i H}\left|\left[-\frac{\alpha}{\alpha+1}\operatorname{grad} p + \frac{1}{c}\,\mathbf{j}\times\mathbf{H}\times\mathbf{H} + \rho_e\mathbf{E}\times\mathbf{H}\right]\right| \ll \\ \ll \left|-\operatorname{grad} p_e + \frac{1}{c}\mathbf{j}\times\mathbf{H} - en\mathbf{E}\right| \qquad (3.2)$$

and, consequently, Ohm's law for a partially ionized gas under condition (3.1) coincides with Ohm's law for a completely ionized gas (2.13) to within the coefficient of the current density.

If the electromagnetic field has a significant effect on the motion of the medium (such problems are known to be of interest from the point of view of magnetohydrodynamics and its applications), the electromagnetic forces are of the order of magnitude of the inertia forces

$$\rho U^2 \approx (n + n_a)\, m_i U^2 = \frac{1}{\alpha}\, n m_i U^2 \approx \frac{1}{c}\, jHL \approx \frac{H^2}{4\pi} \approx p \qquad (3.3)$$

where L and U are a characteristic length and speed in the problem. (It is assumed that convective currents and displacement currents do not exceed in order of magnitude the conductive currents.)

Relation (3.3) shows that for any degree of ionization

$$|\operatorname{grad} p_e| \lesssim \frac{1}{c}\, |\mathbf{j} \times \mathbf{H}| \qquad (3.4)$$

so that grad p_e is insignificant for small degrees of ionization.

If the velocity of the electron gas relative to the ion gas is much smaller than the characteristic speed of the problem

$$U \gg v_e \qquad (3.5)$$

then the following relation holds:

$$\frac{1}{c}\, |\mathbf{j} \times \mathbf{H}| \ll \frac{ne}{c}\, |\mathbf{v} \times \mathbf{H}| \qquad (3.6)$$

Relation (3.5) may be given a very suggestive form using Equation (3.3), namely

$$U \gg \frac{nev_e}{ne} \approx \frac{j}{ne} \approx \frac{1}{\alpha}\, \frac{m_i U^2 c}{eLH}$$

or

$$\frac{1}{\alpha}\, \frac{U}{L}\, \frac{1}{\omega_i} = \frac{1}{\alpha}\, \frac{\Omega}{\omega_i} \ll 1 \qquad \left(\Omega = \frac{1}{T} = \frac{U}{L}\right) \qquad (3.7)$$

We call Ω the characteristic frequency of the problem. Thus condition (3.4) is equivalent to the assumption that the Larmor frequency of the ion is larger than the characteristic frequency of the problem.

If the following inequality holds:

$$U \gg (\varkappa + \varkappa_e)\, v_e, \quad \text{or} \quad \frac{\Omega}{\omega_i} \ll \frac{\alpha}{\varkappa + \varkappa_e} \qquad (3.8)$$

then

$$\frac{\varkappa + \varkappa_e}{c}\, H\, |\mathbf{j} - \rho_e \mathbf{v}| \ll \frac{ne}{c}\, |\mathbf{v} \times \mathbf{H}| \qquad (3.9)$$

With the use of the inequalities (3.1) and (3.7) Ohm's law assumes the form (0.2), usually used in magnetohydrodynamics:

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{1}{c}\, \mathbf{v} \times \mathbf{H}\right) + \rho_e \mathbf{v}, \qquad \sigma = \frac{nec}{H\,(\varkappa + \varkappa_e)} \qquad (3.10)$$

If we determine the time between collisions of the electron (with ions or neutral atoms) according to the formula

$$\frac{1}{\tau^*} = \frac{1}{\tau} + \frac{1}{\tau_e} \tag{3.11}$$

(the collision frequency is equal to the sum of the collision frequencies of the different families), then we obtain for the conductivity the equation

$$\sigma = \frac{ne^2\tau^*}{m_e}$$

agreeing in form with the equation for conductivity of a completely ionized gas.

If in addition to the inequalities (3.1) and (3.7) the inequality (3.8) holds, Ohm's law reduces to the relation

$$\mathbf{E} = -\frac{1}{c}\mathbf{v}\times\mathbf{H} \tag{3.12}$$

This relation is used in the study of the motion of an infinitely conducting medium. Here relation (3.3) permits the inequality $U \gg v_e$ to be introduced in the form given in [8]:

$$\frac{U}{v_e} \approx \frac{neU}{j} \approx \frac{e\alpha HL}{cm_iU} \approx \sqrt{\frac{4\pi\alpha ne^2L^2}{c^2m_i}} \gg 1 \tag{3.13}$$

Under these conditions the inequality (3.1) may be introduced in the form

$$\frac{(1-\alpha)^2}{\varkappa_i} = \frac{2(1-\alpha)^2\tau_i eH}{m_i c} \approx \sqrt{\frac{16\pi(1-\alpha)^4}{\alpha}\,\frac{\tau_i^2e^2U^2n}{m_ic^2}} \ll 1 \tag{3.14}$$

We emphasize that the inequalities (3.13) and (3.14) can be used in place of (3.1) and (3.7) only in consideration of a highly conducting medium, for which the relation (3.8) holds.

The relative magnitude of the terms

$$\frac{1}{c}\mathbf{j}\times\mathbf{H}, \qquad \frac{\varkappa+\varkappa_e}{c}H(\mathbf{j}-\rho_e\mathbf{v})$$

is determined by the number $\omega_e\tau^*$. If

$$\omega_e\tau^* \ll 1 \tag{3.15}$$

then the following relation holds:

$$|\mathbf{j}\times\mathbf{H}| \ll (\varkappa_e+\varkappa)H|\mathbf{j}-\rho_e\mathbf{v}| \tag{3.16}$$

Finally, if the inequality

$$\frac{2(1-\alpha)\tau_i}{\alpha T} \ll 1 \tag{3.17}$$

is satisfied, the following relation holds:

$$\frac{(1-\alpha)^2}{\varkappa_i H}\left|\left\{-\frac{\alpha}{1+\alpha}\operatorname{grad} p\times\mathbf{H}+\frac{1}{c}\,\mathbf{j}\times\mathbf{H}\times\mathbf{H}\right\}\right| \ll \left|-\operatorname{grad} p_e-\frac{en}{c}\,\mathbf{v}\times\mathbf{H}\right| \tag{3.18}$$

Thus the relative magnitudes of the terms in Equation (2.12) that determine the generalized Ohm's law depend on the magnitudes of the following dimensionless parameters:

$$\omega_e\tau^*, \quad \frac{1}{\alpha}\frac{\Omega}{\omega_i}, \quad \frac{(1-\alpha)^2}{\varkappa_i}, \quad \frac{2(1-\alpha)\tau_i}{\alpha T}, \quad \frac{1}{\alpha}\frac{\Omega}{\omega_i}(\omega_e\tau^*)^{-1} \tag{3.19}$$

related to the physical properties of the medium as well as the conditions of the problem under consideration.

The parameter $\omega_e\tau^*$, characterizing the spiral path of the electron between two collisions, determines the relative magnitude of $(\omega_e\tau^*/H)\,\mathbf{j}\times\mathbf{H}$ and $\mathbf{j}$. For atmospheric pressures, temperatures of the order of 10,000° K, and moderate magnetic fields (of the order of 10,000 gauss), this parameter is small, and the term $(\omega_e\tau^*/H)\mathbf{j}\times\mathbf{H}$ can be neglected. However, at these same temperatures and fields, but with pressures of the order of 0.01 atmospheres, the magnitude of $\omega_e\tau^*$ is of the order of unity, and the term $(\omega_e\tau^*/H)\mathbf{j}\times\mathbf{H}$ is significant [9]. Here the phenomenon of "anisotropic conductivity" of the gas appears.

The parameter $\alpha^{-1}\Omega/\omega_i$ determines the relative magnitudes of the terms $(\omega_e\tau^*/H)\mathbf{j}\times\mathbf{H}$ and $(\sigma/c)\mathbf{v}\times\mathbf{H}$. For a heavy gas (argon, air, etc.) at $H \approx 10^4$ gauss, the Larmor frequency of the ion is of the order of $10^6\ \mathrm{sec}^{-1}$, so that for a flow with characteristic speed $U \approx 10^5$ cm/sec with characteristic dimension $L \approx 10$ cm, the quantity $\Omega/\omega_i \approx 10^{-2}$. Consequently, for "pure" gas with thermal ionization the parameter $\alpha^{-1}\Omega/\omega_i$ is large at temperatures below 10,000° K in a broad range of pressures, that is the term $(\omega_e\tau^*/H)\mathbf{j}\times\mathbf{H}$ is considerably larger than $(\sigma/c)\mathbf{v}\times\mathbf{H}$. If under the same conditions the ionization of the gas is increased with a slight amount of ionizing additive, the term $(\omega_e\tau^*/H)\mathbf{j}\times\mathbf{H}$ is insignificant. At very high temperatures this same phenomenon occurs also for "pure" gas.

The parameter $(1-\alpha)^2/\kappa_i$, determining the relative magnitudes of the terms $(\omega_e\tau^*/H)\mathbf{j}\times\mathbf{H}$ and $c^{-1}\mathbf{j}\times\mathbf{H}\times\mathbf{H}$, can be equal to unity only for rarefied gases moving in strong magnetic fields (when not only the electrons but also the ions possess spiral paths). For moderate fields and temperatures this parameter is always much smaller than unity for dense media.

The parameter $[2(1-\alpha)/\alpha]\, r_i/T$, determining the relative magnitudes of $(\sigma/c)\mathbf{v}\times\mathbf{H}$ and $c^{-1}\mathbf{j}\times\mathbf{H}\times\mathbf{H}$, can, in view of condition (2.1), be equal to unity only for very low degrees of ionization of the gas.

The parameter $\alpha^{-1}(\Omega/\omega_i)(\omega_e\tau^*)^{-1}$, being the product of two of the parameters considered above, determines the relative magnitudes of the terms $(\sigma/c)\mathbf{v}\times\mathbf{H}$ and $\mathbf{j}$.

We consider the motion of a dense gas with a moderate magnetic field in the case when the electrons and consequently, in view of (2.10) and (2.11), also the ions do not possess spiral paths

$$\omega_e\tau^* \ll 1, \qquad \varkappa_i \gg 1$$

The inequalities (3.1) and (3.15) hold under these conditions. If the degree of ionization is also significant, that is, (3.7) holds, then (3.17) is satisfied automatically, and Ohm's law takes the form (3.12) with the realization of the inequality

$$\alpha^{-1}(\Omega/\omega_i) \ll \omega_e\tau^* \tag{3.20}$$

(This is the inequality (3.8), transformed by means of (3.11) and Equation (3.10), if the relation $\alpha^{-1}(\Omega/\omega_i) \approx \omega_e\tau^*$ holds. This form of Ohm's law is used in magnetohydrodynamics and is considered above.) If the degree of ionization is also small, so that

$$\frac{1}{\alpha}\,\frac{\Omega}{\omega_i} \gg 1$$

then the inequality inverse to (3.20) holds, and Ohm's law takes the form

$$\mathbf{j} = \sigma\mathbf{E} \tag{3.21}$$

which coincides in form with Ohm's law for a stationary conductor. For low degrees of ionization the induced currents are small, but the currents flowing in the gas must be appreciable if the electromagnetic field affects the motion of the gas, that is, relation (3.3) is satisfied; then naturally currents in the gas can thereby be created only on account of the external electric field. It is evident that under these conditions Ohm's law has the form (3.21). The inequality (3.7) can be violated on account of the reduction of the magnetic field. In a weak magnetic field Ohm's law also has the form (3.21). In this case, if the external electric field is weak, Ohm's law has the form (3.21), but relation (3.3) is violated and the electromagnetic field then does not affect the motion of the medium.

Let the conditions of the problem be such that $\omega_e\tau^* \approx 1$. Then in view of (2.19) and (2.11) $\kappa_i \gg 1$, that is, the inequality (3.1) holds. Under these conditions, if the inequality (3.7) holds, Ohm's law has the form (3.12). If the relation $\alpha^{-1}(\Omega/\omega_i) \approx 1$ holds, then Ohm's law takes the form

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right) - \frac{\omega_e\tau^*}{H}\mathbf{j}\times\mathbf{H} + \frac{\sigma}{ne}\operatorname{grad} p_e \tag{3.22}$$

This form of Ohm's law has been used in a number of papers for the study of the flow of a conducting gas with anisotropic conductivity. Finally, if the inequality inverse to (3.7) holds, Ohm's law has the form

$$\mathbf{j} = -\frac{\omega_e\tau^*}{H}\mathbf{j}\times\mathbf{H} + \frac{\sigma}{en}\operatorname{grad} p_e \quad \text{for } |\mathbf{E}| \lesssim \frac{1}{c}|\mathbf{v}\times\mathbf{H}| \tag{3.23}$$

$$\mathbf{j} = \sigma\mathbf{E} - \frac{\omega_e\tau^*}{H}\mathbf{j}\times\mathbf{H} + \frac{\sigma}{en}\operatorname{grad} p_e \quad \text{for } |\mathbf{E}| > \frac{1}{c}|\mathbf{v}\times\mathbf{H}| \tag{3.24}$$

If the conditions of the problem are such that $\omega_e\tau^* \gg 1$, but the ions still do not possess spiral paths ($\kappa_i > 1$), or the degree of ionization is sufficiently high that the inequality (3.1) is satisfied, then analogous to the preceding case we obtain the following form of Ohm's law:

$$\begin{gathered}
\mathbf{E} = -\frac{1}{c}\mathbf{v}\times\mathbf{H} \quad \text{for } \frac{1}{\alpha}\frac{\Omega}{\omega_i} \ll 1 \\
\sigma\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right) - \frac{\omega_e\tau^*}{H}\mathbf{j}\times\mathbf{H} + \frac{\sigma}{ne}\operatorname{grad} p_e = 0 \quad \text{for } \frac{1}{\alpha}\frac{\Omega}{\omega_i} \approx 1 \\
-\frac{\omega_e\tau^*}{H}\mathbf{j}\times\mathbf{H} + \frac{\sigma}{ne}\operatorname{grad} p_e = 0 \quad \text{for } \frac{1}{\alpha}\frac{\Omega}{\omega_i} \gg 1
\end{gathered} \tag{3.25}$$

Finally, we consider the case when the conditions of the problem are such that the electrons as well as the ions possess spiral paths, and the degree of ionization is so slight that the inequality (3.7) is violated. In this case Ohm's law has the form

for $\dfrac{(1-\alpha)^2}{\varkappa_i} \approx 1, \quad \dfrac{1}{\alpha}\dfrac{\Omega}{\omega_i} \approx 1$

$$\begin{aligned}
ne\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right) + \frac{1}{c}\mathbf{j}\times\mathbf{H} - \frac{\sigma}{ne}\operatorname{grad} p_e - \\
-\frac{(1-\alpha)^2}{H\varkappa_i}\left\{-\frac{\alpha}{\alpha+1}\operatorname{grad} p\times\mathbf{H} + \frac{1}{c}\mathbf{j}\times\mathbf{H}\times\mathbf{H}\right\} = 0
\end{aligned} \tag{3.26}$$

for $\dfrac{(1-\alpha)^2}{\varkappa_i} \approx 1, \quad \dfrac{1}{\alpha}\dfrac{\Omega}{\omega_i} \ll 1$

$$\mathbf{E} = -\frac{1}{c}\mathbf{v}\times\mathbf{H}$$

for $\dfrac{(1-\alpha)^2}{\varkappa_i} \approx 1, \quad \dfrac{1}{\alpha}\dfrac{\Omega}{\omega_i} \gg 1$

$$-\operatorname{grad} p_e + \frac{1}{c}\mathbf{j}\times\mathbf{H} - \frac{(1-\alpha)^2}{H\varkappa_i}\left\{-\frac{\alpha}{1+\alpha}\operatorname{grad} p + \frac{1}{c}\mathbf{j}\times\mathbf{H}\times\mathbf{H}\right\} = 0 \tag{3.27}$$

if $|\mathbf{E}| \lesssim c^{-1} |\mathbf{v} \times \mathbf{H}|$; if this relation is not satisfied the **term** $\sigma\mathbf{E}$ is added to (3.27). If

$$\frac{(1-\alpha)^2}{\varkappa_i} \gg 1$$

then Ohm's law has the following forms:

$$\text{for } \frac{1}{\alpha}\frac{\Omega}{\omega_i} \ll 1, \quad \frac{2(1-\alpha)\tau^i}{\alpha T} \approx 1$$

$$-ne\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{H}\right) - \frac{(1-\alpha)^2}{\varkappa_i H}\left\{-\frac{\alpha}{\alpha+1}\operatorname{grad} p + \frac{1}{c}\mathbf{j} \times \mathbf{H} \times \mathbf{H}\right\} = 0 \qquad (3.28)$$

$$\text{for } \frac{1}{\alpha}\frac{\Omega}{\omega_i} \ll 1, \quad \frac{2(1-\alpha)\tau_i}{\alpha T} \gg 1, \quad |\mathbf{E}| \lesssim \left|\frac{\mathbf{v}}{c} \times \mathbf{H}\right|$$

$$-\frac{\alpha}{1-\alpha}\operatorname{grad} p + \frac{1}{c}\mathbf{j} \times \mathbf{H} \times \mathbf{H} + \rho_e \mathbf{E} \times \mathbf{H} = 0 \qquad (3.29)$$

For $\alpha^{-1}\Omega/\omega_i \gtrsim 1$ Ohm's law has the form (3.29).

We note in conclusion that in carrying out the estimates the characteristic quantities L, U, T were everywhere assumed to be equal for all mechanical and electromagnetic variables. These estimates are therefore inapplicable to flows with various kinds of boundary and transition layers, and to other problems in which the characteristic magnitudes of various variables may be different. In these problems (as in establishing any problem) it is necessary to carry out estimates analogous to the preceding ones in order to choose a form of Ohm's law. Furthermore, in all the estimates it was essential to use the assumption that the electromagnetic field significantly affects the motion of the medium, that is, relation (3.3) holds, which determines the form of one of the basic parameters $\alpha^{-1}\Omega/\omega_i$. If the conditions of the problem are such that relation (3.3) is violated (motion of a weakly conducting medium at high speed or slow motion of a highly conducting medium), it is necessary in the estimates to use the inequality $U \gg v_e$ directly, and not (3.7), which is a consequence of $U \gg v_e$ only under the conditions (3.3).

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